4.4

In Summary

Key Ideas

- There are fewer permutations when some of the objects in a set are identical compared to when all the objects in a set are different. This is because some of the arrangements are identical.
- The number of permutations of n objects, where a are identical, another b are identical, another c are identical, and so on, is

$$P = \frac{n!}{a!b!c!\ldots}$$

For example, in the set of four objects a, a, b, and b, the number of different permutations, P, is

$$P = \frac{4!}{2! \cdot 2!}$$

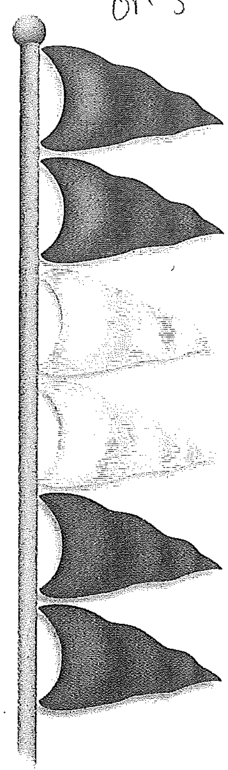
$$P = 6$$

The six different arrangements are *aabb*, *bbaa*, *abab*, *baba*, *abba*, and *baab*.

Need to Know

- Dividing $n!$ by $a!$, $b!$, $c!$ and so on deals with the effect of repetition caused by objects in the set that are identical. It eliminates arrangements that are the same and that would otherwise be counted multiple times.

Quiz tomorrow on §4.1-4.3



CHECK Your Understanding

1. Evaluate the following expressions.

a) $\frac{7!}{3! \cdot 2!}$ $= 7 \cdot 5 \cdot 4 \cdot 3 = 420$ b) $\frac{8!}{2! \cdot 2! \cdot 2!} = 7!$ $= 5040$ c) $\frac{10!}{4! \cdot 3! \cdot 2!}$ $= 12600$ d) $\frac{12!}{2! \cdot 4! \cdot 5!}$ $= 83160$ ✓

2. How many different signals can be made from 6 flags hung in a vertical line, using 2 identical white flags, 2 identical red flags, and 2 identical blue flags? $\frac{6!}{2!2!2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2}} = 90$ ✓

3. Six nickels are flipped simultaneously. How many ways can three coins land as heads and three coins land as tails?

$\frac{6!}{3!3!} = \frac{\cancel{6} \times 5 \times 4 \times \cancel{3!}}{3! \cancel{3!}} = 20$ ✓

PRACTISING

4. A hockey team has a record of 10 wins, 5 losses, and 3 ties in 18 games. How many different ways could this record have occurred?

$\frac{18!}{10!5!3!} = 2450448$ ✓

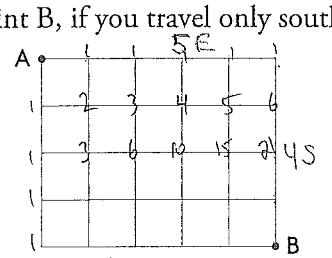
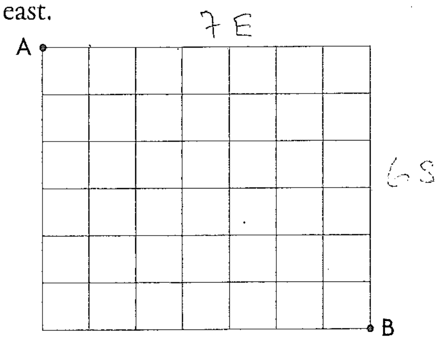
5. Norm bought 3 chocolate-chip cookies, 2 peanut-butter cookies, and 4 oatmeal cookies from the corner bakery to give to his 9 grandchildren. How many ways can he distribute 1 cookie to each grandchild?

$\frac{9!}{3!2!4!} = 1260$ ✓

6. How many different arrangements can be made using all the letters in each word?
   a) YUKON $5! = 120$
   b) ALBERTA $\frac{7!}{2!} = 2520$
   c) MANITOBA $\frac{8!}{2!} = 20160$
   d) SASKATCHEWAN $\frac{12!}{3!2!} = 39916800$ ✓

7. A clerk at a bookstore is restocking a shelf of best-selling novels. He has five copies each of three different novels.
   a) How many different ways can he arrange the books on the shelf? $\frac{15!}{5!5!5!} = 756756$ ✓
   b) How many different ways can these books be arranged on the shelf if the copies of the same novel must be grouped together? $= 3! = 6$

8. Create a counting problem that can be solved using the expression $\frac{8!}{2! \cdot 4!}$.

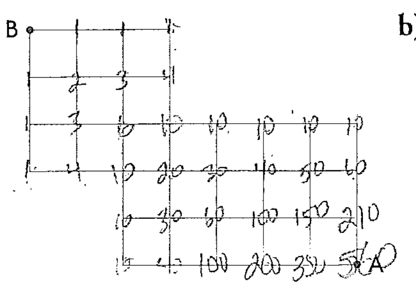
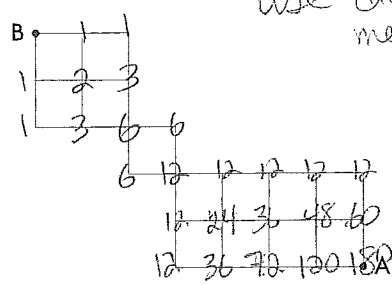
9. Determine the number of routes there are to get from point A to point B, if you travel only south or east.
   a) 
   $\frac{9!}{5!4!} = 126$
   b) 
   $\frac{13!}{7!6!} = 1716$ ✓

10. Jess always walks to her friend's house, which is eight blocks north and five blocks west of her house. How many different routes can she take if she always walks either north or west?

$\frac{13!}{8!5!} = 1287$ ✓

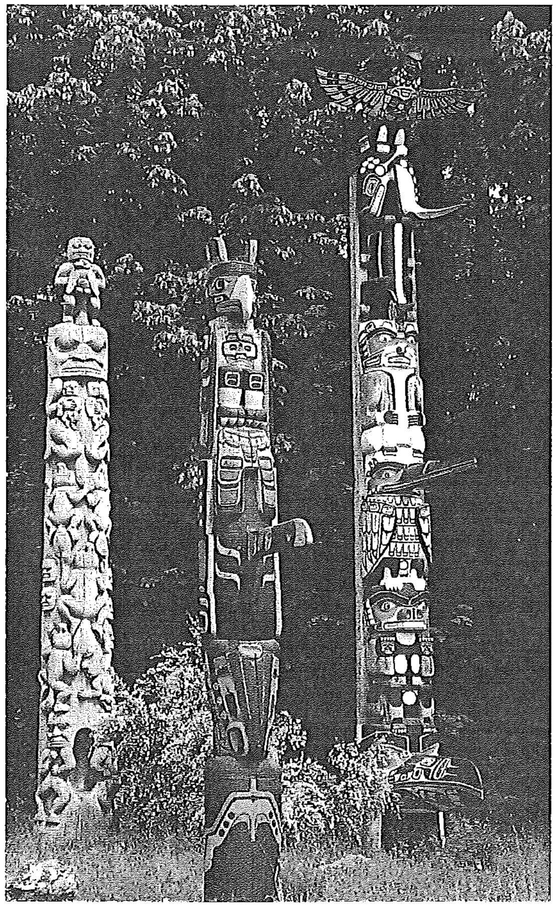
11. How many different routes are there from A to B, if you travel only north or west?

use diagram method

   a) 
   560
   b) 
   180 ✓

12. A true–false test has eight questions. How many different permutations of answers can the teacher create if five answers are true and three answers are false? $\frac{8!}{5!3!} = 56$

13. The Chief Wakas Totem Pole, located in Vancouver's Stanley Park, represents the talking stick and characters in an Owikeno (Kwakiutl) story. In the photograph below, the Chief Wakas Totem Pole is on the right. The figures, from the top down, are Thunderbird, Killer Whale, Wolf, Wise One, Huxwhukw (a mythical bird), Bear, and Raven.



The original Chief Wakas Totem Pole (a replica is pictured, far right) was built as a ceremonial entrance to Chief Wakas's house in Alert Bay, Vancouver Island.

a) Suppose a new totem pole is created using different permutations of all seven figures. How many different arrangements of these figures on the pole are possible? $7! = 5040$

b) Suppose another new totem pole is to include these seven figures: two of Thunderbird and two of Wolf, as well as Killer Whale, Wise One, and Bear. How many ways can these figures be arranged on a totem pole? $\frac{7!}{2!2!} = 1260$

$_nP_n = \frac{n!}{(n-n)!}$

14. Explain why $_nP_n = n!$ cannot be used to determine the number of arrangements of a group of n items when there are a identical items in the group and $a < n$. you will be double counting due to the identical items

15. a) How many ways can 9 coins (3 dimes, 4 quarters, and 2 loonies) be arranged in a line? State any assumptions you are making.

$\frac{9!}{3!4!2!} = 1260$ assuming each group are identical coins ✓

b) What if the line must begin and end with a loonie? State any assumptions you are making.

$\frac{7!}{3!4!} = 35$ same assumption

16. Sam, Nunzio, and 8 of their friends are playing outside on a hot summer day. How many ways can 10 freezies (3 grape, 2 lime, and 5 orange) be distributed among the 10 children if Sam must have lime and Nunzio must have grape?

Sam lime, Nunzio's grape

$\frac{8!}{\underset{\text{grape}}{2!}\underset{\text{orange}}{5!}} = 168$ ✓

17. How many permutations are possible using all the letters of the word STATISTICS for each condition described below? ← instead of 10 since A and C are restricted

10 letters

a) You must start with A and end with C.

$\frac{8!}{\underset{S}{3!}\underset{T}{3!}\underset{I}{2!}} = 560$

b) The two I's must be together.

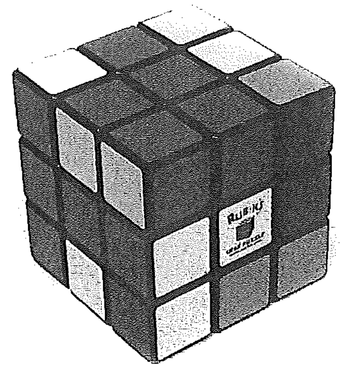
$\frac{9!}{\underset{S}{3!}\underset{T}{3!}} = 10080$ ← treat I's as 1 letter since must be together ✓

Closing

18. Consider the words BANANAS and BANDITS. Explain why there are 12 times the number of permutations possible using all the letters of BANDITS compared to the number of permutations possible using all the letters of BANANAS.

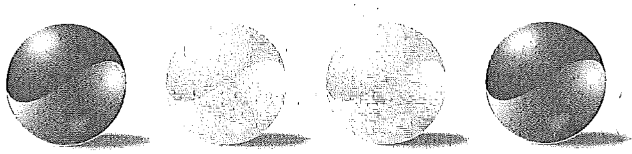
Extending

19. A Rubik's cube can be thought of as a grid in three dimensions. How many routes are there from the top rear vertex of the cube to the lower front vertex of the cube, if each route must be as short as possible and follow the grid lines?



20. How many ways can 20 soccer players on a travelling team be assigned to hotel rooms for each situation?
    a) There are only 10 double rooms.
    b) There are 5 quadruple rooms.

21. A bag contains three identical red marbles and three identical white marbles. Four marbles are drawn out of the bag and arranged in a row from left to right.
    a) How many different arrangements might be made?
    b) What is the likelihood that the arrangement is, from left to right, red, white, white, red?



4.5

Combinations

$_nC_r = \frac{n!}{r!(n-r)!}$

order doesn't matter

In Summary

Key Ideas

- When order does not matter in a counting problem, you are determining combinations. For example, *abc*, *acb*, *bac*, *bca*, *cab*, and *cba* are the six different permutations of the letters *a*, *b*, and *c*, but they all represent the same single combination of letters.
- When all *n* objects are being used in each combination, there is only one possible combination.

Need to Know

- From a set of *n* different objects, there are always fewer combinations than permutations when selecting *r* of these objects where $r \leq n$. For example, the number of permutations, *P*, possible using two of the three letters *a*, *b*, and *c* is

$$P = \frac{3!}{(3-2)!}$$
$$P = 6$$

The six permutations are *ab*, *ba*, *ac*, *ca*, *bc*, and *cb*. However, of these six permutations, only *ab*, *ac*, and *bc* are different two-letter combinations.

Permutations

$_nP_r = \frac{n!}{(n-r)!}$

order does matter

FURTHER Your Understanding

cans	dry goods	f&v
B	R	L
B	L	R
R	B	L
R	L	B
L	B	R
L	R	B

1. Brian, Rachelle, and Linh volunteer at the food bank on Saturdays.

a) In the morning, each person is needed for a different job: stacking cans, stocking dry goods, and cleaning fruits and vegetables. In how many different ways can they be chosen for these jobs? order matters

$_3P_3 = \frac{3!}{0!} = 6$

b) List all the ways the three volunteers can be assigned the jobs in part a).

c) In the afternoon, all three volunteers are needed to unload vehicles arriving from food drives. In how many ways can they be chosen for this job? order doesn't matter $_nC_r = \frac{3!}{3!0!} = 1$

d) In the situations above, which involved permutations and which involved combinations? Explain how you know. (a)+(b) (c)

2. Explain the main difference between the following:

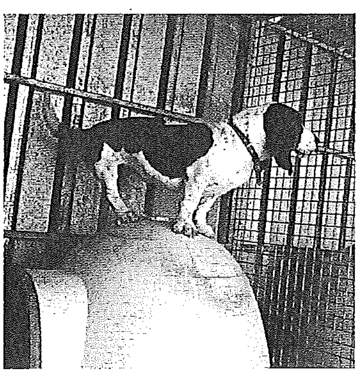
order matters — more options

- Permutations of 4 objects out of a group of 6 different objects $\frac{6!}{2!} = 360$
- Combinations of 4 objects out of a group of 6 different objects $\frac{6!}{4!2!} = 15$

order doesn't matter — fewer options

3. There are 10 members of student council. How many ways can 4 of the members be chosen to serve on the dance committee? $_{10}C_4 =$ $\frac{10!}{[illegible]} = 210$

order doesn't matter



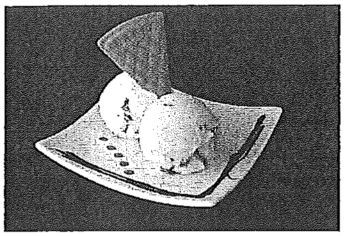
4. There are 12 dogs at the local animal shelter. To increase the likelihood that the dogs will be adopted, 3 of them will appear on a TV morning show. How many ways can 3 of the 12 dogs be selected to appear?

Order of the 3 dogs doesn't matter

$_{12}C_3 = \frac{12!}{3!9!} = 220$

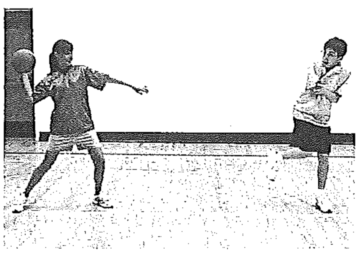
4.6

CHECK Your Understanding



1. Joe has a choice of four flavours of ice cream for his two-scoop sundae: vanilla, strawberry, chocolate, and butterscotch.
   a) List all the permutations for a two-flavour sundae. (order) ${}_4P_2 = \frac{4!}{2!} = 12$
   b) List all the combinations for a two-flavour sundae. (no order) ${}_4C_2 = \frac{4!}{2!2!} = 6$
   c) How is the number of two-flavour permutations related to the number of two-flavour combinations? Explain. ÷2!, smaller since each combination can be written 2 ways

2. From a group of five students, three students need to be chosen for a car-wash committee. order doesn't matter
   a) How many committees are possible? ${}_5C_3 = \frac{5!}{3!2!} = 10$
   b) How many committees are possible, if only two students are needed on the committee? ${}_5C_2 = \frac{5!}{3!2!}$
   c) Compare your answers for parts a) and b). What do you notice? Explain why this occurred. Same



3. How many ways can 6 people be selected from a group of 12 to form a dodge-ball team? if order doesn't matter ${}_{12}C_6 = \frac{12!}{6!6!} = 924$

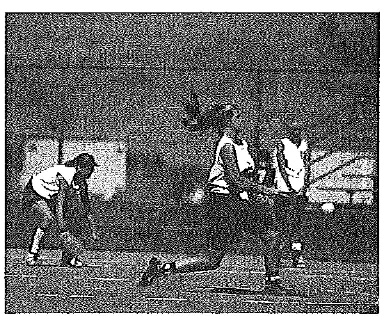
PRACTISING

4. Evaluate the following.
   a) ${}_5C_3 = \frac{5!}{3!2!} = 10$
   b) ${}_9C_8 = \frac{9!}{8!1!} = 9$
   c) $\binom{6}{4} = \frac{6!}{4!2!} = 15$
   d) ${}_{10}C_0 = \frac{10!}{0!10!} = 1$
   e) $\binom{12}{6} = \frac{12!}{6!6!} = 924$
   f) ${}_8C_1 = \frac{8!}{1!7!} = 8$

5. How many ways can 6 players be chosen to start a volleyball game from a team of 10? ${}_{10}C_6 = \frac{10!}{6!4!} = 210$

6. An online music store offers 5 free songs when you join. It has 55 hip-hop songs available. How many different combinations of hip-hop songs can you download for free?

7. The card game Crazy Eights is played with a standard deck of playing cards. How many different 8-card hands can be dealt? ${}_{52}C_8 = \frac{52!}{8!44!}$



8. Connie's softball team has 15 players. How many ways can the coach choose his starting lineup of 9 players, if Connie must be the pitcher?
   a) Does this problem involve permutations or combinations? Explain. rest of team
   b) Solve the problem. ${}_{14}C_8 = \frac{14!}{8!6!} = 3003$

9. a) Marnie claims that $\binom{6}{2} = \binom{6}{4}$. Do you agree? Justify your decision. $= \frac{6!}{2!4!} = 15$ True
   b) Examine several more cases with the same relationship as part a). What do you notice? equal always
   c) Based on your observations in part b), suggest a relationship between $\binom{n}{r}$ ~~and~~ = $\binom{n}{n-r}$ for the natural numbers n and whole numbers r.

10. Suppose that 5 teachers and 8 students volunteered to be on a graduation committee. The committee must consist of 2 teachers and 3 students. How many different graduation committees does the principal have to choose from?

T S

$_5C_2 \times {_8C_3}$

$\frac{5!}{2!3!} \times \frac{8!}{3!5!} = 560$ ✓

11. How many 5-person committees can be formed from a group of 6 women and 4 men, under each of the following conditions?

a) There are no conditions.
b) There must be exactly 3 women.
c) There must be exactly 4 men.
d) There can be no men.
e) There must be at least 3 men.

12. A youth hostel has 3 rooms that contain 5, 4, and 3 beds, respectively. How many ways can 12 students be assigned to these rooms?

$_{12}C_5 \times {_7C_4} \times {_3C_3}$

$\frac{12!}{5!7!} \times \frac{7!}{4!3!} \times \frac{3!}{3!0!} = 27720$ ✓

13. a) For each expression, state the number of different objects in the set and how many are used in each combination.

i) $_nC_r = \frac{5!}{3!(5-3)!}$ $n=5$, $r=3$ **ii)** $_nC_r = \frac{10!}{2!(10-2)!}$ $n=10$, $r=2$ **iii)** $_nC_r = \frac{5!}{3!2!}$ $n=5$, $r=3$

b) Choose one expression from part a) and create a combination problem that could be solved using that expression.

14. Pascal's triangle can be created using the combinations below.

a) Evaluate each combination.
i) $_0C_0$ = 1
ii) $_1C_0$, $_1C_1$ = 1, 1
iii) $_2C_0$, $_2C_1$, $_2C_2$ 1, 2, 1
iv) $_3C_0$, $_3C_1$, $_3C_2$, $_3C_3$ 1, 3, 3, 1 ✓
v) $_4C_0$, $_4C_1$, $_4C_2$, $_4C_3$, $_4C_4$ 1, 4, 6, 4, 1

b) Copy the triangular arrangement of boxes at right. Write each answer from part a) in a box in order, starting with the answer to part i) at the top.

c) In parts a) and b), you created the first five rows of Pascal's triangle. Describe at least two patterns you observe in the triangle.

d) Write two more rows of Pascal's triangle using the patterns you observed.

e) How does Pascal's triangle relate to the pathway problems in Lesson 4.4?

$\boxed{1}$
$\boxed{1}$ $\boxed{1}$
all ones $\boxed{1}$ $\boxed{2}$ $\boxed{1}$ sum
$\boxed{1}$ $\boxed{3}$ $\boxed{3}$ $\boxed{1}$ sum
$\boxed{1}$ $\boxed{4}$ $\boxed{6}$ $\boxed{4}$ $\boxed{1}$
1 5 10 10 5 1
1 6 15 20 15 6 1

15. Solve each equation. State any restrictions on the variable.

a) $_nC_2 = 15$ $n \geq 2$ **c)** $4\binom{n}{2} = \binom{n+2}{3}$ see paper ✓

b) $_nC_4 = 35$ **d)** $\binom{6}{r} = 15 = 5 \times 3$ use logic

$0 \leq r \leq 6$

a) $_nC_2 = 15 = \frac{n!}{2!(n-2)!}$

$15 = \frac{n(n-1)\cancel{(n-2)!}}{2!\cancel{(n-2)!}}$

$2 \times 15 = n^2 - n$

$0 = n^2 - n - 30$ $n = 6$ or $\cancel{-5}$

$(n-6)(n+5)$

d) $\binom{6}{r} = 15 = \frac{6!}{r!(6-r)!}$

$15 = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2!(6-2)!}$

$15 = \frac{6 \times 5 \times 4 \times 3}{4 \times 3 \times 2 \times 1}$

$r = 2$ or $r = 4$

pg 281 Ma 12

15c

$$4\binom{n}{2} = \binom{n+2}{3}$$

$$\overset{2}{\cancel{4}}\,\frac{n!}{\cancel{2}!\,(n-2)!} = \frac{(n+2)!}{3!\,(n+2-3)!}$$

$$2\,\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = \frac{(n+2)!}{6\,(n-1)!}$$

$$2(\cancel{n})(n-1) = \frac{(n+2)(n+1)(\cancel{n})\cancel{(n-1)!}}{6\,\cancel{(n-1)!}}$$

$$12(n-1) = (n+2)(n+1)$$

$$12n-12 = n^2+n+2n+2$$

$$0 = n^2-9n+14$$

$$= (n-2)(n-7)$$

$$n = 2 \text{ or } 7$$

check

$n=2$

$$4 = \frac{4!}{3!(1)!} = 4 \checkmark$$

$n=7$

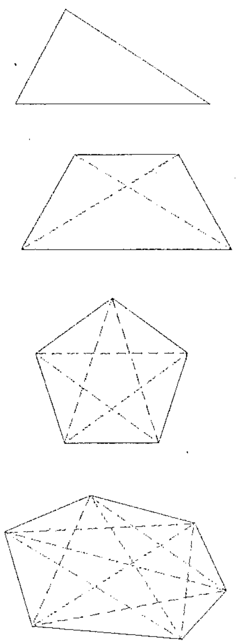
$$\overset{2}{\cancel{4}}\cdot\frac{7!}{\cancel{2}!(5!)} = \frac{9!}{3!6!}$$

$$84 = \frac{9\cdot8\cdot7}{6} = 8$$

✓

16. A children's hospital in a city of about one million people is running a charity lottery called Lucky Six to raise money. Players choose six numbers from the numbers 1 to 66. The player wins if the six numbers chosen match six numbers drawn at random by the organizers. order doesn't matter

a) How many ways could the player win? 1 way to win
b) How many ways could the player lose? $_{66}C_6 - 1 = 90{,}858{,}767$
c) Is this a reasonable game for the hospital to run? Explain. 90 million ways to lose
no only 1 million people!



17. How can the combination formula be used to determine the number of diagonals in an n-sided polygon?

18. There are 7 boys and 13 girls in the school art club. A group of 5 is needed to set up an art exhibit. How many different groups of 5 students with at least 2 boys are there to choose from?
a) Solve the problem using direct reasoning.
b) Solve the problem using indirect reasoning.
c) Which approach do you prefer? Explain why.

Closing

19. a) How are combinations and permutations similar? How are they different? Use examples in your answers.
b) If you know the value of $_nP_r$, how can you determine the value of $_nC_r$? Use examples in your answer.

Extending

20. A CD player holds five different CDs. The CD player is set on shuffle so it randomly selects songs to play from the five CDs. This chart shows the number of songs there are on each CD.

CD Number	1	2	3	4	5
Number of Songs	12	14	15	12	18

Determine the probability that each of the following events will happen during the first five songs played.
a) The five songs will be from CD 2 and CD 4.
b) One of the five songs will be from each CD.
c) Your favourite song from each of the 5 CDs will be played.

21. Simplify: $\binom{n}{3} + \binom{n}{2} + \binom{n}{1}$

22. Prove: $_{n+1}C_r = {_nC_r} + {_nC_{r-1}}$

4.7

CHECK Your Understanding

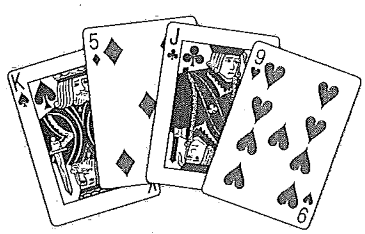
1. Identify whether each situation involves permutations or combinations. Explain how you know.
   a) Choose 3 toppings for a pizza from 25 different possibilities. C – no order
   b) Choose a CEO, president, and vice-president from a group of 20 candidates. P order matters
   c) Determine the number of outcomes possible when rolling 3 dice: 1 red, 1 blue, and 1 white. P order matters
   d) Determine the number of ways 5 children from a group of 11 can start in a game of pickup basketball. C order doesn't matter
   P if position matters

2. Consider two situations:
   - Situation A: A committee of 3 is to be selected from a group of 10 people. C order doesn't matter.
   - Situation B: An executive committee consisting of a president, a vice-president, and a secretary is to be selected from 10 people. P

   Determine which of these situations involves combinations and which involves permutations. Explain your answer.

3. Maddy arrived at an auction, and there were only 8 items left to bid on. She likes all 8 items but especially likes 3 items. She can afford to have winning bids on only 3 items. How many ways can she bid on 3 items under each of the following conditions?
   a) She bids on only her 3 favourite items. 1 way (all 3) to bid on
   b) She bids on any 3 of the 8 items. $_8C_3 = \frac{8!}{3!5!} = 56$ ways to randomly choose 3 items from 8

PRACTISING



4. From a standard deck of 52 cards, how many different four-card hands are there with one card from each suit? $_{13}C_1 \times {_{13}C_1} \times {_{13}C_1} \times {_{13}C_1} = \left(\frac{13!}{1!12!}\right)^4 = 2$[illegible]

5. How many ways can the top five cash prizes be awarded in a lottery that sold 200 tickets
   a) if each ticket is not replaced when drawn? $_{200}P_5 = \frac{200!}{195!}$ $200\times199\times198\times197\times196 = 3\times10^{[illegible]}$
   b) if each ticket is replaced when drawn? $200\times200\times200\times200\times200 = 3.2\times10^{11}$

6. How many ways can the 5 starting positions on a basketball team (1 centre, 2 forwards, and 2 guards) be filled from a team of 2 centres, 4 guards, and 6 forwards? $_2C_1 \times {_6C_2} \times {_4C_2} = \frac{2!}{1!1!} \times \frac{6!}{2!4!} \times \frac{4!}{2!2!} = 180$

7. How many ways can five different pairs of identical teddy bears be arranged in two rows of five for a photograph? $\frac{10!}{2!2!2!2!2!} = 113400$

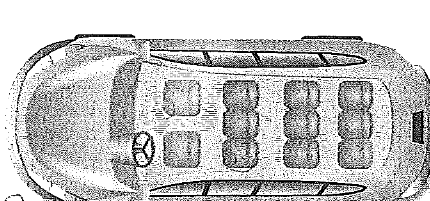
8. Five different signal flags fly on the flag pole of a coast guard ship. You can send signals using one or more of the flags. How many different signals can be sent using at least three of these flags? order matters for signals!

$$_5P_3 + {_5P_4} + {_5P_5}$$
$$\frac{5!}{2!} + \frac{5!}{1!} + \frac{5!}{0!} = 60 + 120 + 120 = 300$$

9. Six different types of boats have pulled into a marina and want to dock at the six available slips. The six slips are adjacent to each other. How many ways can the six boats dock so that the two cabin cruisers, which are travelling together, are docked next to each other?

$2! \cdot 5! = 240$

10. Mark is the coach of a boys' basketball team. He has rented a passenger van as shown to drive to a weekend tournament. How many ways can the 10 players sit in the van if Mark drives and there must be three players on each bench? State any assumptions you are making. order matters $_{10}P_{10} = 10! = 3628800$



if order didn't matter then $_{10}C_{10} = \frac{10!}{10!0!} = 1$ way

11. a) How many different arrangements are possible for the letters in the word FUNNY if there are no conditions? $\frac{5!}{2!} = 60$

b) How many different arrangements are possible if each arrangement must start and end with an N? N 3! N $= 6$

12. A combination lock opens when the right sequence of three numbers from 0 to 99 is used. The same number may be used more than once. How many sequences are there that consist entirely of odd numbers?

order matters
50 odd #'s $50 \times 50 \times 50 = 125000$

13. How many different routes can you take to get to a location five blocks south and six blocks east, if you travel only south or east?

$\frac{11!}{5!6!} = 462$

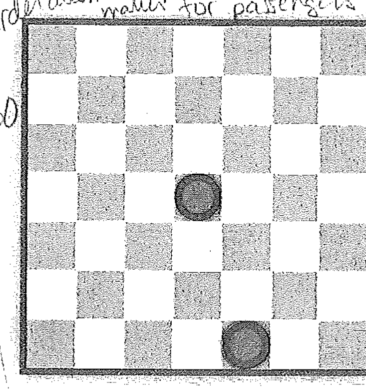
14. Three vehicles are taking 16 musicians to a concert: a 5-person car driven by Joe, a 4-person car driven by Kuami, and a 7-passenger van driven by Angela. How many ways can the 16 people be assigned to the 3 vehicles?

order doesn't matter for passengers

$_{13}C_4 \times {}_9C_3 \times {}_6C_6 = \frac{13!}{4!9!} \times \frac{9!}{3!6!} \times \frac{6!}{6!0!} = 60060$

15. On the game board shown, the red checker is allowed to move toward the top of the board diagonally left or right. If the black checker is encountered, the red checker cannot move into its square or jump over it. Determine the number of paths the red checker can follow from its starting position to any yellow square along the top of the board.

top of board



start

if order of passengers matters
$_{15}P_4 \times {}_{10}P_3 \times {}_6P_6$
$= \frac{15!}{11!} \times \frac{10!}{7!} \times \frac{6!}{1!}$
$= 16982700000$

16. How many different five-card hands that contain at most three hearts can be dealt from a standard deck of playing cards?

Closing

17. Create a flow chart that you could use to help you make decisions about how to solve counting problems. — done handout

#14 key
$_{16}C_5 \times {}_{11}C_4 \times {}_7C_7$
$\frac{16!}{5!11!} \times \frac{11!}{4!7!} \times \frac{7!}{7!0!}$
$= 144144 0$ — but implys that doesn't matter who drives but it does matter

4 Chapter Self-Test

1. A manufacturer uses a six-character serial number for a line of products. The first and second characters are upper-case letters (A to Z). The third, fourth, and fifth characters are digits (0 to 9). There are only three choices for the last position: A, B, and X.
   a) How many different serial numbers are possible, if repetition of characters is allowed?
   b) How many different serial numbers are possible, if no repetition is allowed?

a) $26 \times 26 \times 10 \times 10 \times 10 \times 3 = 2028000$ ✓

b) $25\ 24\ 10\ 9\ 8\ 3 = 1296000$ ✓

2. How many different ways are there to draw 1 card that is a spade or a diamond from a standard deck of 52 cards?

$_{13}C_1 + {}_{13}C_1 = \left(\frac{13!}{1!\,12!}\right)2 = 26$ ✓

spades diamonds

3. Simplify each expression. State the restrictions on the variable.
   a) $(n + 10)(n + 9)!$ $= (n+10)!$ $n \geq -9$
   b) $\frac{(n-2)!}{n!}$ $= \frac{(n-2)!}{n(n-1)(n-2)!} = \frac{1}{n^2-n}$ $n \geq 2$ ✓

key wrong for restriction. 3a) $n \geq -9$

4. A parking lot attendant has 5 cars to park: 1 blue, 1 white, 1 red, and 2 black.
   a) How many different ways can the 5 cars be parked side by side?
   b) How many different ways can the cars be parked so the 2 black cars are next to each other?

a) $5! = 120$ (if " " " diff) key

b) $2! \times 4! = 48$ ✓

5. A book club offers a selection of four books from a list of nine different titles.
   a) How many different four-book selections can be made? $_9C_4 = \frac{9!}{4!\,5!} = 126$ ✓
   b) How many different four-book selections can be made if the four selections are listed in order of preference? $_9P_4 = \frac{9!}{5!} = 3024$ ✓
   c) Why are the answers to parts a) and b) different? Explain. order matters in part b

6. Solve for n: $_nP_4 = 84(_nC_2)$

$\frac{n!}{(n-4)!} = 84\left(\frac{n!}{2!(n-2)!}\right) \rightarrow (n-2)(n-3) = 42$ $n \geq 2$

$n^2 - 3n - 2n + 6 - 42 = 0$

$n^2 - 5n - 36 = 0$

$(n-9)(n+4) = 0$

$n = 9$ ✓

7. David and Susan belong to a math club at their school. There are 6 boys and 8 girls in the club. How many different ways can a 5-person committee be selected from the 14 club members under each of the following conditions?
   a) There must be 2 boys and 3 girls. $_6C_2 \times {}_8C_3 = \frac{6!}{2!\,4!} \times \frac{8!}{3!\,5!} = 840$ ✓
   b) There must be at least 2 boys. $_6C_2 \times {}_8C_3 + {}_6C_3 \times {}_8C_2 + {}_6C_4 \times {}_8C_1 + {}_6C_5 \times {}_8C_0 = 1526$ ✓ (840, 560, 120, 6)
   c) David and Susan must be on the committee. $1 \times 1 \times {}_{12}C_3 = 220$
   d) There must be more girls than boys. $_6C_2\,{}_8C_3 + {}_6C_1\,{}_8C_4 + {}_6C_0\,{}_8C_5 = 1316$ (840, 420, 56)

8. How many different arrangements are there of the letters in the word TEETH? $\frac{5!}{2!\,2!} = 30$ ✓

9. The nine members, five boys and four girls, of a softball team are arranging themselves in a line for a team photograph. For one of the poses, the photographer wants a boy on either side of each of the four girls. How many different arrangements are possible? alternate

BGBGBGBGB

$5!\,4! = 2880$ ✓

WHAT DO You Think Now? Revisit **What Do You Think?** on page 227. Have your answers and explanations changed?

Ch4 Review

Q: **How do you decide what strategy to use to solve a counting problem?**

A: First, determine if order is important. If it is, use a permutation model. If not, use a combination model.

- Look for conditions. Consider these first as you develop your solution.
- If there is a repetition of r of the n objects to be eliminated, it is usually done by dividing by $r!$.
- If a problem involves multiple tasks that are connected by the word AND, then the Fundamental Counting Principle can be applied: multiply the number of ways that each task can occur.
- If a problem involves multiple tasks that are connected by the word OR, the Fundamental Counting Principle does not apply: add the number of ways that each task can occur. This typically is found in counting problems that involve several cases.

could have:

2) Q T L — Heads, Tails — H H H, H H T, H T H, etc

Study Aid

- See Lesson 4.6, Example 3, and Lesson 4.7, Examples 1 to 3.
- Try Chapter Review Questions 17 to 19.

PRACTISING

Lesson 4.1

1. When is the Fundamental Counting Principle used to solve a counting problem? Use an example in your explanation.

diff tasks related by "AND"
ie how many ways to wear 1 of 3 shorts and one of 5 shirts.

2. Create a tree diagram to show all the possible ways that three coins can land when you flip a quarter, a toonie, and a loonie all at the same time.

3. A student is writing a 10-question multiple-choice test. Each question has 4 choices: A to D. How many different sets of answers can the student give?

$4\times4\times4\times4\times4\times4\times4\times4\times4\times4 = 4^{10} = 1048576$

Lesson 4.2

4. Solve each equation. State the restrictions on the variable.

a) $\dfrac{(n+2)!}{n!} = 20$ $\quad n \geq 0$

$\dfrac{(n+2)(n+1)\cancel{n!}}{\cancel{n!}} = 20$

$n^2+3n-18=0$

$(n-3)(n+6)=0$

$n=3$ or $\cancel{-6}$

b) $\dfrac{(n+1)\cdot n!}{(n-1)!} = 132$ $\quad n \geq 1$

$\dfrac{(n+1)n(\cancel{n-1})!}{(\cancel{n-1})!}$

$= n^2+n-132=0$

$(n-11)(n+12)$

$n=11$, $\cancel{-12}$

5. Which expression has a larger value: ${}_6P_6$ or $\dfrac{8!}{6!}$? Explain how you know.

${}_6P_6 = 6!$ $\qquad \dfrac{8!}{6!} = 56$

6. Chorale Saint-Jean from Edmonton, Alberta, is the largest and most active francophone choir in Western Canada. If the singers have rehearsed 12 different songs for an upcoming tour, in how many different orders could they perform the 12 songs?

${}_{12}P_{12} = 12! = 479001600$

Lesson 4.3

7. How many different ways can a director of education, a superintendent of curriculum, and a superintendent of finance be selected from group of 25 candidates? order matters $_{25}P_3 = \frac{25!}{22!} = 13800$

8. A website offers an online practice driving test that consists of 10 questions selected from a bank of 25 questions. The level of each question ranges from easy to difficult. How many different ways can the test be created in each of the following situations? order matters

a) There are no conditions. $_{25}P_{10} = \frac{25!}{15!} = 1.186 \times 10^{13}$

b) The easiest question of the 25 is always first and the most difficult question is always last. $1 \times {}_{23}P_8 \times 1 = \frac{23!}{15!}$

9. Suppose you draw five cards from a standard deck of cards and arrange them in a row from left to right in the order you draw them. How many different five-card arrangements are possible? order matters

$_{52}P_5 = \frac{52!}{47!} = \frac{52!}{(52-5)!}$

Lesson 4.4

10. Consider the 11 letters in the word MATHEMATICS. How many different arrangements are possible in the following situations? 2M's 2A's 2T's

a) All the letters are used. $\frac{11!}{2!2!2!} = 4989600$

b) All the letters are used, but each arrangement must begin with the C. $1 \times \frac{10!}{2!2!2!} = 453600$ omit

11. Tina is playing with a tub of building blocks. The tub contains 3 red blocks, 5 blue blocks, 2 yellow blocks, and 4 green blocks. How many different ways can Tina stack the blocks in a single tower in each situation below? 14 blocks

a) There are no conditions. $\frac{14!}{3!5!2!4!} = 2522520$

b) There must be a yellow block at the bottom of the tower and a yellow block at the top. $1 \times \frac{12!}{3!5!4!} \times 1 = 27720$

Lesson 4.5

12. Which expression results in the greatest value: $_{10}C_5$ or $_{11}C_7$ or $\binom{15}{2}$?

$\frac{10!}{5!5!} = 252$ $\quad \frac{11!}{7!4!} = 330$ $\quad \frac{15!}{2!13!} = 105$

choosing larger # of the set

13. How many different selections of 4 books can Ruth choose from a box of 20 different books at her neighbour's garage sale? order doesn't matter $_{20}C_4 = \frac{20!}{4!16!} = 4845$

14. Liz claims that if you have a set of n different objects and you select r of them, the number of combinations you can make will always be greater than the number of permutations. Do you agree? Justify your decision. No, with P order matters, so there are more ways to arrange

Lesson 4.6

15. The town council is forming a committee, and 9 men and 10 women have volunteered. How many different ways can a committee of 4 people be chosen in each situation below? no order

a) There are no conditions. $\binom{19}{4} = \frac{19!}{4!15!} = 3876$

b) There must be an equal number of men and women on the committee. $\binom{9}{2}\binom{10}{2} = \frac{9!}{2!7!} \times \frac{10!}{2!8!} = 1620$

c) No men can be on the committee. $\binom{10}{4} = \frac{10!}{4!6!} = 210$

16. How many different ways can 15 teachers be divided into 3 groups of 5 for an activity at a staff meeting? no order $_{15}C_5 \times {}_{10}C_5 \times {}_5C_5$ $\frac{15!}{5!10!} \times \frac{10!}{5!5!} \times 1 = 756756$

Lesson 4.7

17. If 12 points are arranged in a circle, how many different ways can the points be joined to form straight lines? order not impt $_{12}C_2 = \frac{12!}{2!10!} = 66$

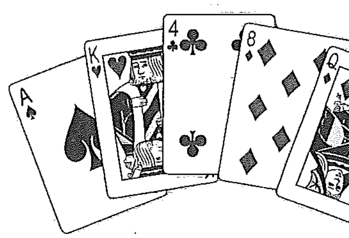
18. Kim, Kandice, and Kerry are female triplets. They and their 10 cousins are posing for a series of photographs. One pose involves all 13 children. How many ways can the 7 boys and 6 girls be arranged in one row under each of the following conditions? BGBGBGBGBGBGB

a) The boys and girls must alternate positions. $7!6! = 3628800$

b) The triplets must stand next to each other. $3! \times 10 = 11! = 39916800$

19. How many different five-card hands with at least two face cards can be dealt from a standard deck of playing cards?

all hands of 5 − 1 face card − 0 face card



$= \frac{52!}{5!47!} - \frac{40!}{4!36!} \cdot \frac{12!}{1!11!} - \frac{40!}{5!35!}$

$= 2598960 - 1096680 - \ldots$

$= 844272$